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TE RAUHITANGA TAIAO

Lake Ototoa Study: Modelling Thermal Stratification

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Lake Ototoa study: modeling thermal stratification

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1 Executive Summary

This report describes the application of a computer model to predict daily vertical temperature profiles in Lake Ototoa over a seasonal cycle from 18 March 2006 through 8 May 2007. The main goal of the application was to determine whether such computer model predictions could be carried out successfully enough to provide the basis for longer term simulations to examine effects of climate change on mixing and stratification in the lake.

The computer model utilizes information on lake bathymetry, daily meteorological data and daily inflow and outflow data. It accounts for all major physical processes that cause thermal stratification and vertical mixing within a lake or reservoir. The basic output from the model is a series of daily temperature profiles.

The simulations used meteorological data collected from a weather station installed by Auckland Regional Council (ARC) near the lake. Inflows and outflows were estimated from a water balance for the lake. Model predictions have been compared against temperatures measured in the lake by ARC as part of their routine monitoring of the lake, and against temperatures recorded continuously by thermistors on a mooring installed by ARC in the lake.

Significant difficulties were encountered in the model application that involved uncertainties associated with the meteorological and inflow data. Gaps in the meteorological record were filled by regression against data from an automated weather station at Dargaville.

The model accurately captures the overall patterns of stratification and mixing in terms of timing, strength of stratification and depth of the thermocline. Results for maximum, minimum and mean temperatures predicted at the moored thermistor depths also agreed well with measured temperatures.

The quality of the simulations will clearly enable the model to be used to investigate effects of climate change or as a base for coupled hydrodynamic – water quality modeling. The limiting factor in either case is the availability of input data (mostly long term meteorological data for a study of climate change effects) to run this newly established model.

2 Introduction

This report describes the application of a computer model to predict daily vertical temperature profiles in Lake Ototoa over a seasonal cycle. The cycle includes winter mixing; warming through spring and summer with accompanying temperature stratification; and cooling in autumn leading to overturn and a return to mixing over the entire depth in winter (Green 1975, Gibbs 2006). The development and application of the model was to proceed concurrently with field work and reporting on water quality in the lake carried out by Max Gibbs of NIWA Hamilton under a contract between NIWA and Auckland Regional Council (ARC). The contract requests "… the development of a hydrodynamic model of the lake using 'DYRESM' to determine how the lake is likely to have responded over time to climatic variability. This modelling will be developed using historical meteorological data and will be refined using data collected from the ARC weather station near Lake Ototoa and the in-lake instrumentation during the present study."

Although the model has been developed and applied successfully, it was not possible to achieve all the objectives related to historical data and response to climate variability for two reasons – problems with finding a site with suitable historical data, and problems in using data measured by the weather station installed by ARC near the lake. With regard to the availability of suitable historical data, an exhaustive search of climate stations nearest to Lake Ototoa in consultation with NIWA meteorologist Andrew Harper showed that the closest site with a somewhat similar coastal exposure, that recorded all the parameters required by the model over a long enough period to examine climate variability, was at Dargaville approximately 72 km north of Lake Ototoa. It was felt that this distance was too great for the Dargaville data to be used in the initial development and application, without first establishing the applicability of the Dargaville data to Lake Ototoa, and moreover that the initial development and application of the model should be based on the meteorological data measured closer to the lake by the ARC's weather station. However, this also proved difficult because of problems experienced in obtaining a continuous record from the ARC weather station. Considerable efforts were devoted to quantifying correlations between the Dargaville data, to filling in gaps in the data measured by the ARC weather station, and to assembling a continuous data set that could be used as input for the model.

Additional work was required to calculate a water balance for the lake so that daily inflow volumes, required to run the model, could be estimated.

Meaningful correlations have now been established between the Lake Ototoa weather station and Dargaville, which led to the successful development and application of the lake model. The simulations covered the period for which meteorological data were available from the ARC weather station near the lake, 18 March 2006 through 8 May 2007. Model predictions have been compared against temperatures measured in the lake by ARC during their (approximately) monthly visits to the lake, and as recorded hourly by thermistors on a mooring installed by ARC in the lake. Locations of the weather station and thermistor mooring are shown in Figure 1.

Data available to run the model and to compare with model predictions only span a single complete season. There are not enough data for separate “calibration” and “verification” runs over separate seasons. This is not a serious problem, however, since the model is essentially a physics-based one and there is limited flexibility for varying model parameters to achieve a fit to observations.

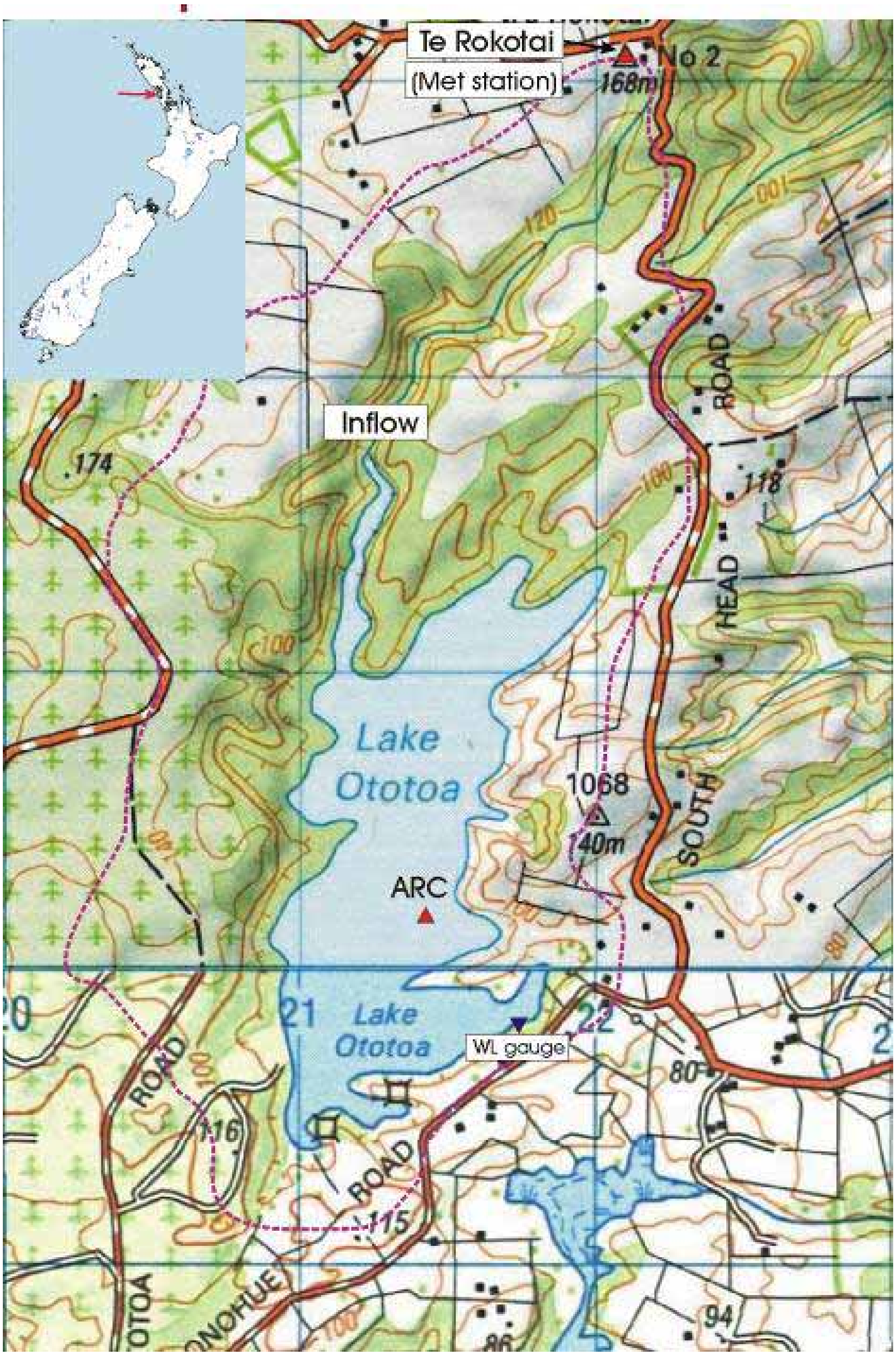
The main difficulties encountered in the model application involved uncertainties in the input data required to run the model. These included gaps in the meteorological data; differences between the weather conditions at the meteorological station compared to those over the lake; and lack of time-series data for inflow (streamflow and groundwater) volumes and temperatures.

The following sections contain descriptions of the meteorological data (Section 3), the model (Section 4), the study site (Section 5), the input data used for the simulation (Section 6), and the results (including comparisons with observations) (Section 7). The potential for extending the modeling to longer-term simulations and/or to include water quality is considered in Section 8.

This report focuses on the thermal regime of the lake and does not address questions related to oxygen, nutrient or phytoplankton dynamics; these were considered in the earlier Lake Ototoa water quality study of Gibbs (2006).

Figure 1

Map showing locations of Lake Ototoa (see inset, upper left corner), Te Rokotai weather station (red triangle at top of map), ARC thermistor chain mooring (red triangle in the lake labeled ARC), ARC water level gauge (blue triangle on southeast arm), and the only inflowing stream of importance for the modeling.



3 Meteorological data

The lake model requires data for incoming solar radiation, wind speed, air temperature and humidity, rainfall, and either incoming longwave (infrared) radiation from the atmosphere or fractional cloud cover (which is then used by the model together with air temperature to calculate incoming longwave radiation). These must be supplied by the user as daily averages, or as averages over a shorter time interval (e.g., hourly) if available, for every day of the simulation period. The quality of the simulations depends on the accuracy of the meteorological data and on how well they represent conditions above the water surface of the lake. This section describes work that was necessary to assemble a good input meteorological dataset before modelling could begin.

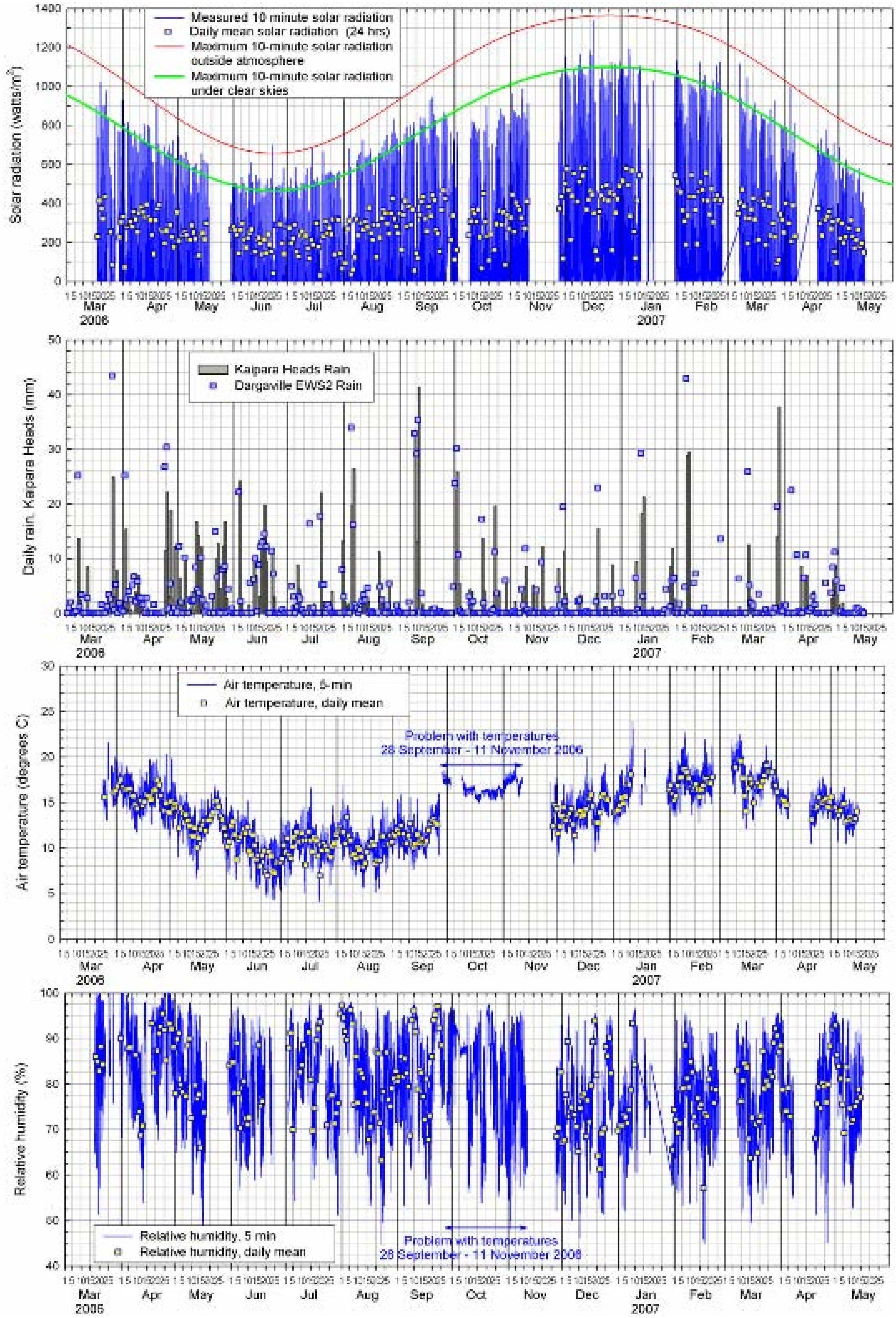
The nearest permanent climate station with similar coastal exposure and with continuous records of all required meteorological parameters is at Dargaville, approximately 72 km north of Lake Ototoa. To obtain meteorological data more representative of conditions over the lake, ARC installed a weather station on Te Rokotai, a hill 1.74 km from the northern shore of the lake, at altitude 168 m ASL (location shown in Figure 1; also see photo in Figure 2 below). The station has sensors to measure incoming solar radiation, wind speed and direction, air temperature and humidity, and barometric pressure. Data for all variables have been measured since 18 March 2006. The sampling interval is 5 minutes, except for solar radiation, which is measured (mostly) at 10 minute intervals. Rainfall data used for modeling are from the ARC-operated rain gauge Kaipara Heads at Wallers (site 644211, map reference Q 9:213292, NZMG coordinates 2621300E, 6529200N).The full set of data from Te Rokotai and Kaipara Heads are plotted in Figure 3.

Figure 2

Te Rokotai weather station; photo by ARC

Figure 3

Meteorological data from Te Rokotai weather station near Lake Ototoa, except for rainfall, which is from Kaipara Heads and Dargarville.



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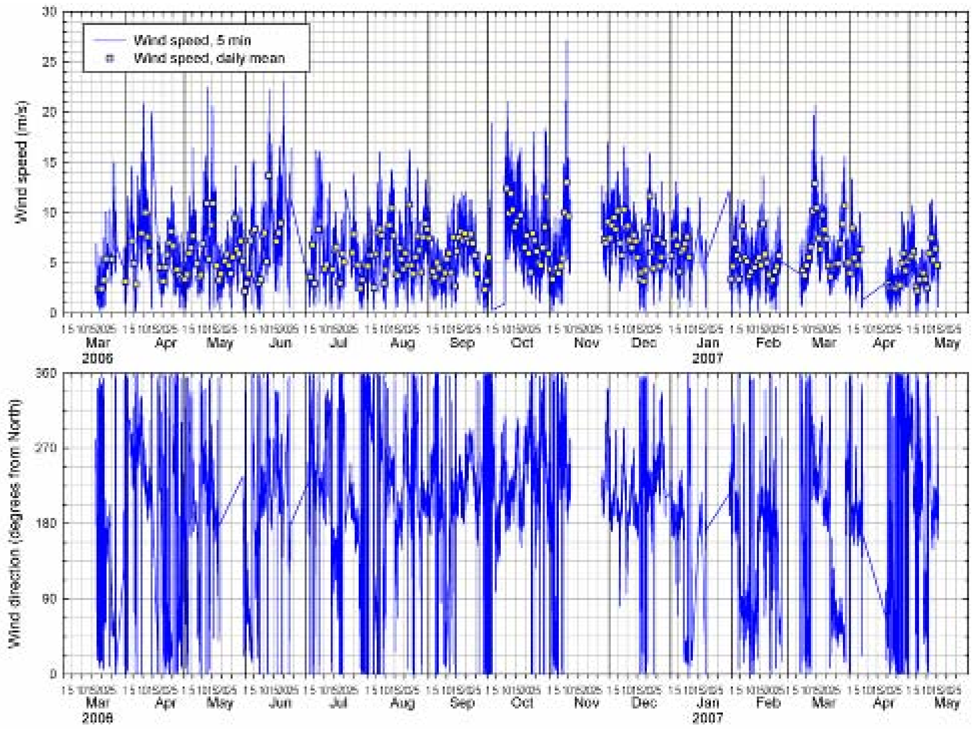
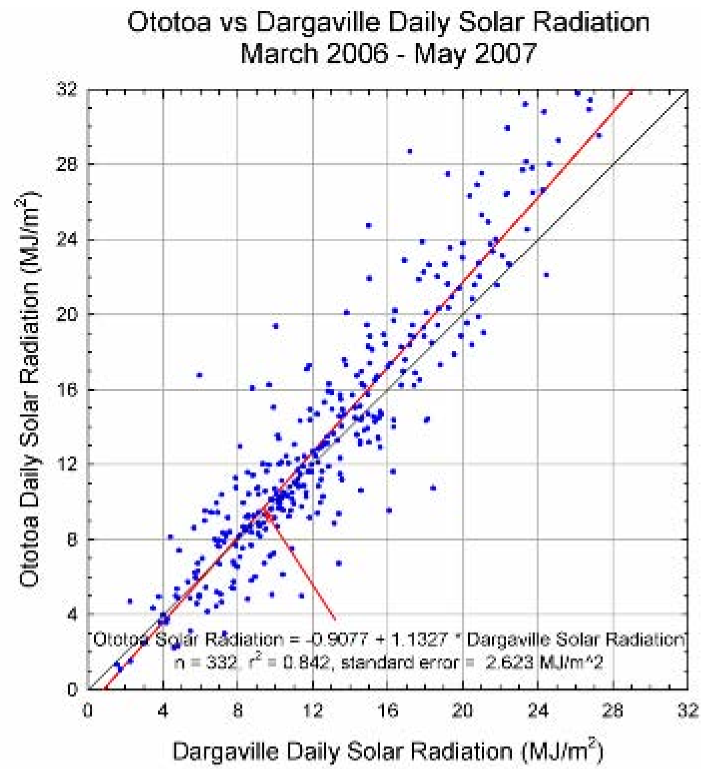


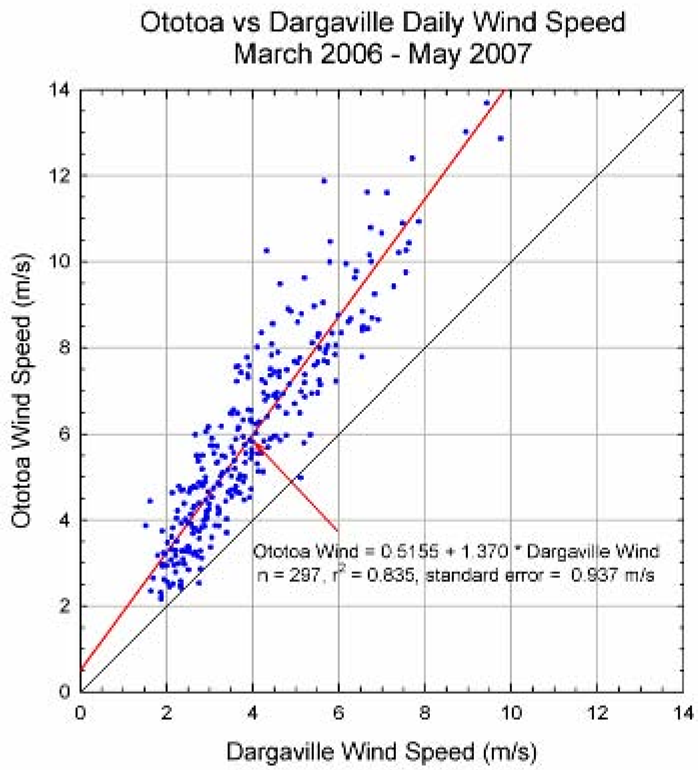
Obtaining reliable and continuous measurements from the Te Rokotai weather station proved difficult, and there are gaps in the records of all parameters. Gaps range from 10 minutes to 16 days; the longer gaps can be seen in the plots of Figure 3. In the case of air temperature the longest gap was extended by a further 44 days of poor quality data. Gaps less than 5 hours were filled in by linear interpolation. Longer gaps were filled by regression against Dargaville data. Regressions based on daily means were used to develop the model input dataset. These gave better correlations than those based on hourly values. The scatter plots and regression equations for all correlations are shown Figure 4 for daily means, and in Figure 5 for hourly means (wind speed, air temperature and humidity only). The values of the regression coefficients, correlation coefficients and standard errors are summarised in Table 1.

All regressions for daily means give reasonably good correlations, except for rainfall. Fortunately, there were no gaps in the ARC Kaipara Heads rainfall record for the period of the simulation, so no correlation/regression with Dargaville was needed.

Figure 4

Correlations and regression equations for daily meteorological data measured at Te Rokotai weather station versus data from Dargaville electronic weather station.





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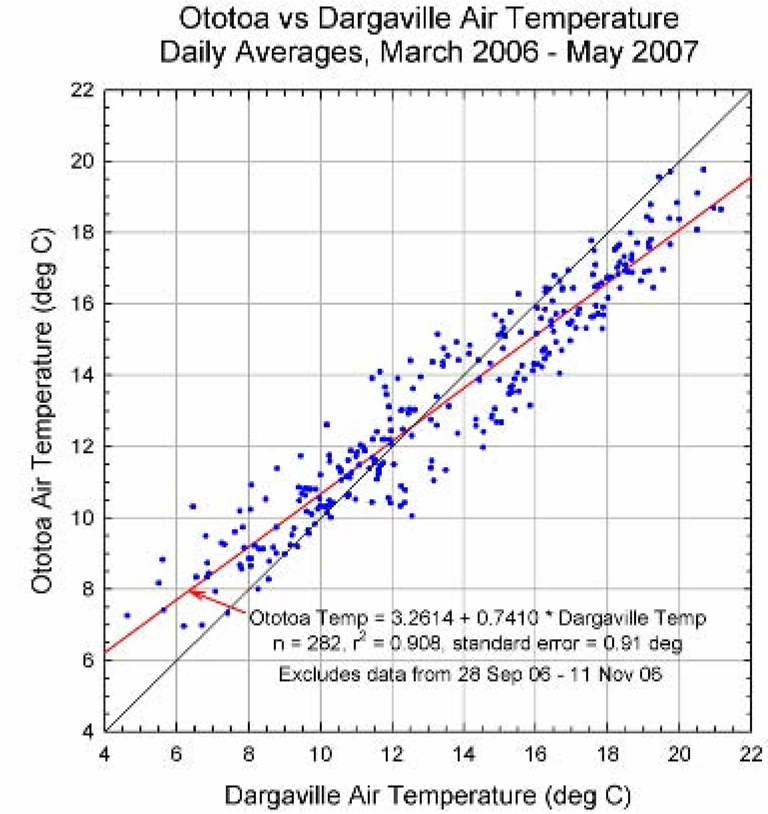

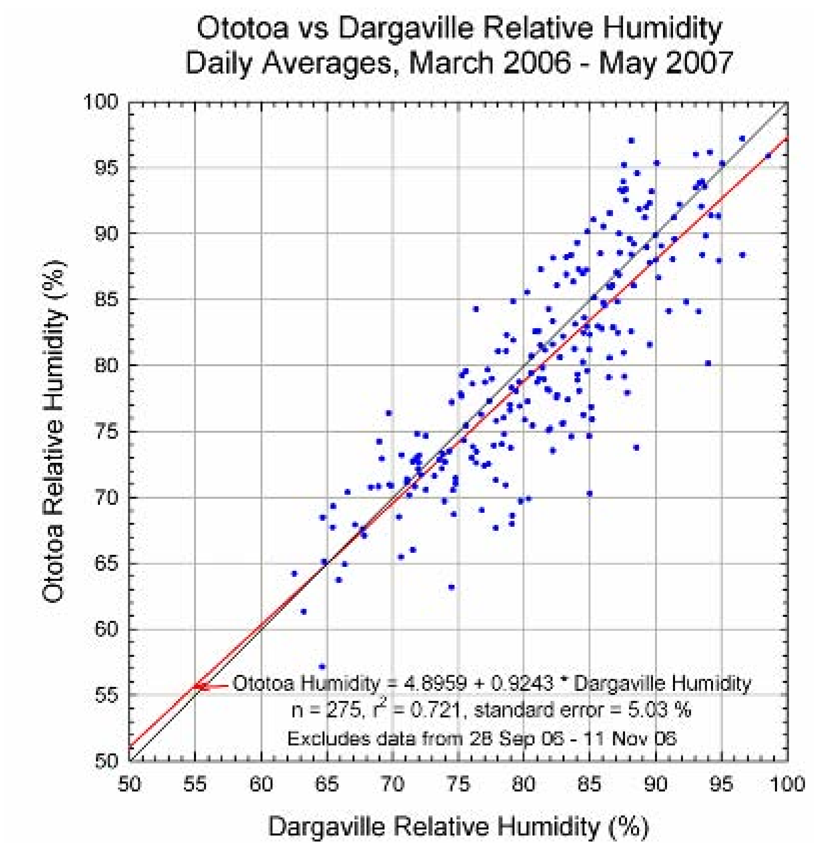

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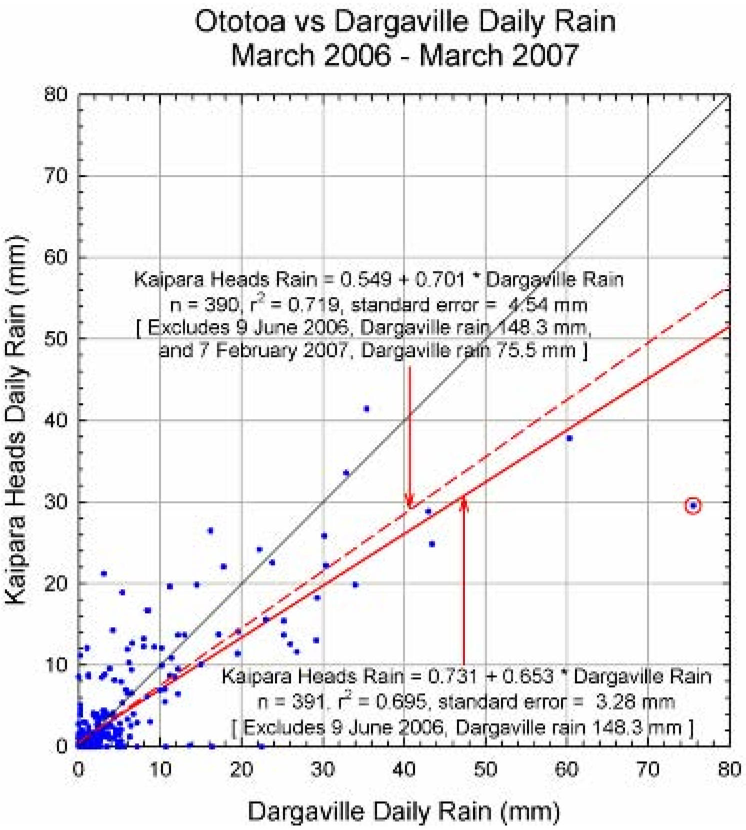
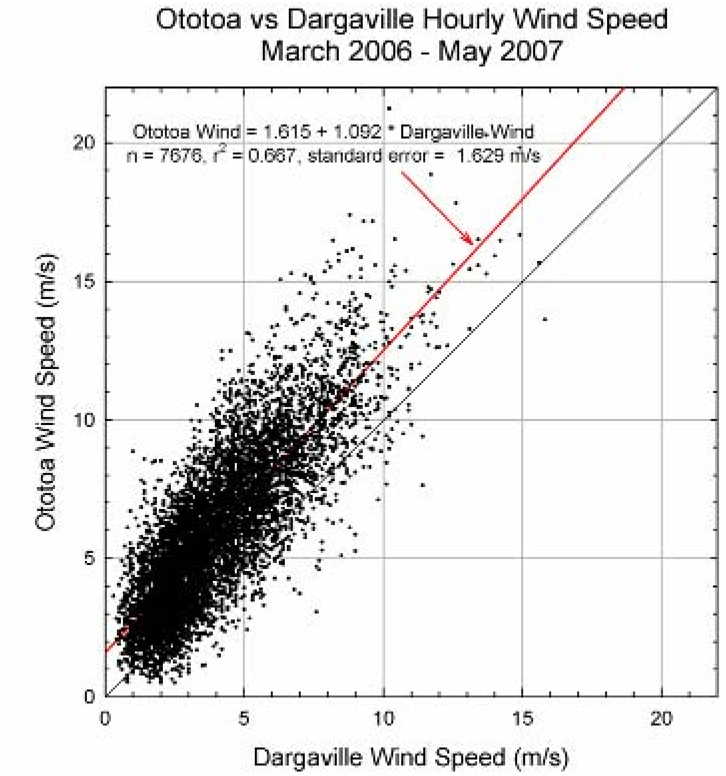
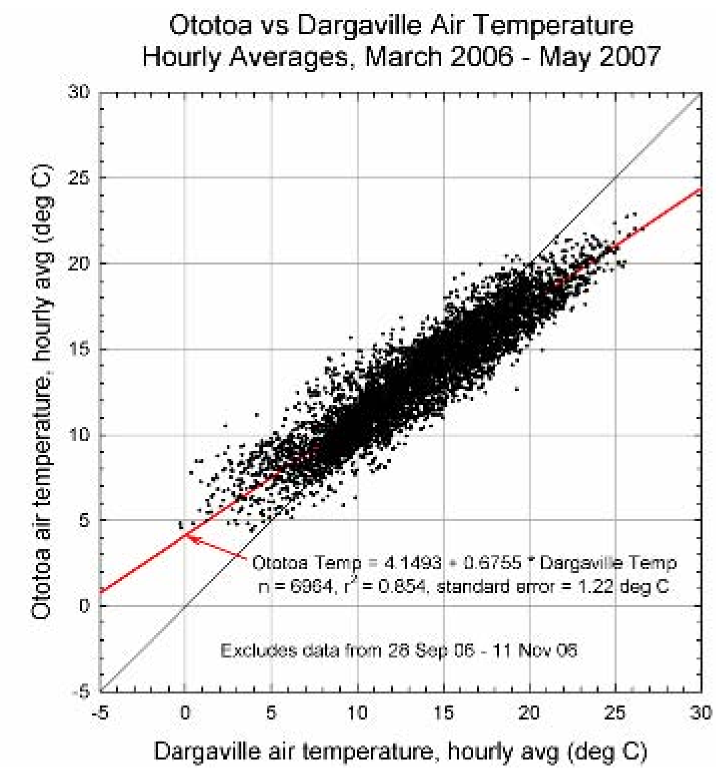


Figure 5

Correlations and regression equations for hourly meteorological data measured at Te Rokotai weather station versus data from Dargaville electronic weather station.





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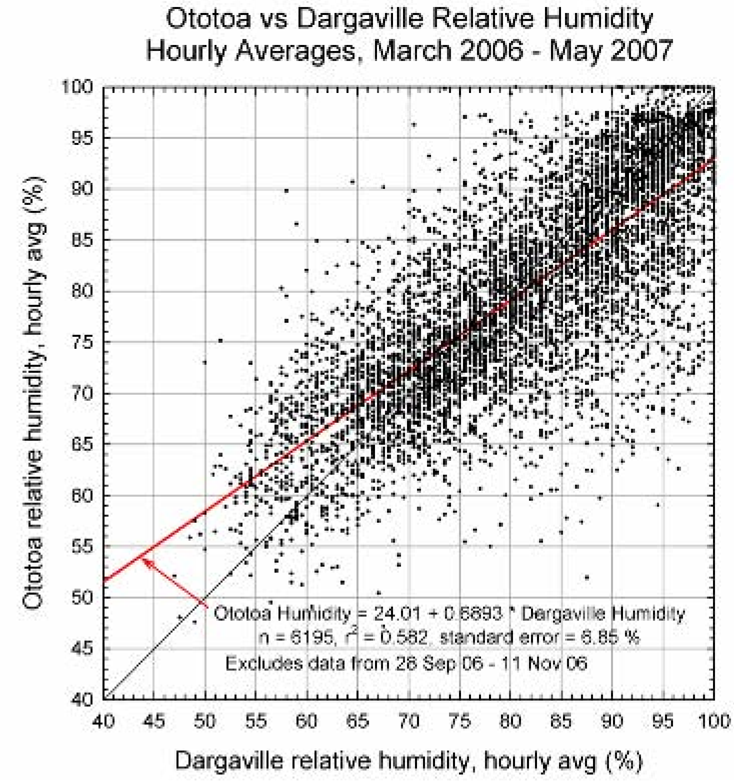


Table 1

Regressions of Lake Ototoa meteorological data on Dargaville meteorological data						
Period of overlap: 18 March 2006 - 8 May 2007						
L Ototoa = b0 + b1 x Dargaville						
	Solar radiation	**Wind speed**	**Air temperature***	**Relative humidity***	**Kaipara Heads vs Dargaville Rain****	**Kaipara Heads vs Dargaville Rain*****
	Daily totals (MJ/m^2)	**Daily averages (m/s)**	**Daily averages (deg C)**	**Daily averages (%)**	**Daily totals (mm)**	**Daily totals (mm)**
b[0]	-0.9077	0.5155	3.2614	4.8959	0.7308	0.5495
b[1]	1.1327	1.3697	0.7410	0.9243	0.6528	0.7005
r ²	0.842	0.835	0.908	0.721	0.695	0.719
n	332	297	282	275	391	390
Std error	2.623	0.937	0.910	5.028	3.281	4.540
		Hourly averages (m/s)	**Hourly averages (deg C)**	**Hourly averages (%)**		
b[0]		1.6147	4.1493	24.010		
b[1]		1.0920	0.6755	0.6893		
r2		0.667	0.854	0.582		
n =		7676	6964	6195		
Std Error		1.629	1.224	6.851		
* Excludes data from 28 September - 11 November 2006						
** Excludes large value for Dargaville on 9 June 2006 (148.3 mm)						
*** Excludes large values for Dargaville on 9 Jun 06 (148.3 mm) and 7 Feb 07 (75.5 mm)						

4 Model description

In this study we used the University of Western Australia – Centre for Water Research's computer model DYRESM (Dynamic Reservoir Simulation Model) that predicts temperature profiles in lakes and reservoirs. The model has been widely applied in Australia, New Zealand and overseas, and has been documented in the scientific literature by Imberger (1979) and Imberger and Patterson (1979, 1990), who give overviews and further references. Further documentation can also be found at http://www.cwr.uwa.edu.au/ with links to "services" and "models". The model utilizes information on lake bathymetry (areas within depth contours; volumes below depth contours; outlet elevation(s)); daily meteorological data (incoming solar radiation, cloud cover, wind speed, air temperature, humidity, rainfall); and daily inflow and outflow data. It accounts for all major physical processes that cause thermal stratification and vertical mixing within a lake or reservoir.

The model is referred to as "one-dimensional" because it only simulates vertical variations in water properties. The basic output from the model is a series of daily temperature profiles (and profiles of salinity and density if required). The profiles are meant to be representative of conditions averaged over the horizontal extent of the lake. Hence the model is applicable to lakes in which vertical variations are generally much more pronounced than horizontal variations. These tend to be deeper lakes of relatively small horizontal extent. Lake Ototoa fits well in this category, as can be seen from the generally horizontal contours of temperature, conductivity and oxygen in summer transects plotted by Gibbs (2006).

DYRESM has a unique system for dividing the water column into layers so that it can model temperature stratification; this distinguishes it from most other one-dimensional lake models. The layers are not tied to a fixed numerical depth grid, but are free to move up and down in the water column and to adjust their thicknesses in response to mixing, inflow and outflow processes. This so-called "Lagrangian" layering system, in which the layers can move vertically in response to vertical water movements, provides distinct numerical advantages over fixed-grid systems in terms of computational stability and eliminating effects of artificial numerical diffusion. It also allows for increased resolution to be provided in areas where thermal gradients are steep (e.g., the thermocline), with decreased resolution where gradients are small (e.g., the epilimnion and hypolimnion). This improves computational efficiency.

An initial temperature profile must be specified by the user at the start of a model run; in this study daily mean temperatures for 18 March 2006 measured by the ARC thermistor chain at 3 m intervals were used. This date corresponds to the beginning of the meteorological dataset. The model steps through the input meteorological and inflow data one day at a time. At the start of each day the model divides the water column into layers at a resolution set by the user (20 cm was used for Lake Ototoa). This resolution needs to be fine enough to accurately model solar radiation attenuation within the water column, turbulent mixing in the epilimnion, evolution of the thermocline, and the intrusion of inflowing waters from streams or groundwater at their levels of neutral buoyancy in the lake. For reservoirs, the layer thicknesses must also be able to resolve processes associated with managed outflows drawn from several depths.

The model simulates inflow and outflow on a daily basis, but has a sub-daily loop with a time step set by the user to simulate heat, mass and momentum exchange between the lake and the atmosphere and the consequent heating and mixing processes that occur in the lake. One hour is typical for the sub-daily time step, and this was used for Lake Ototoa. The model can use meteorological data supplied at time intervals less than a day. This was not done for Lake Ototoa because of the gaps in the data from the Te Rokotai weather station and the necessity of using data from Dargaville, which is discussed in more detail below.

In some lakes, salinity variations (as concentration of total dissolved solids) make an important contribution to density variations. In most New Zealand lakes, salinity variations are small and their contribution to density variations is negligible in comparison with effects of temperature, and salinity can be neglected. We have assumed this to be the case for Lake Ototoa, based on specific conductance data presented by Gibbs (2006, Figure 16). His measurements show that while variations in conductivity are large enough to be detected and are consistent with other measurements of lake water chemistry, they are small enough for salinity to act as a passive tracer for different water masses (e.g., oxygenated versus anoxic) and to have little impact on density compared to temperature.

5 Study site

Lake Ototoa (latitude 36.51°S, longitude 174.24°E) is located in sand dunes on the west coast of the North Island, near the northern end of the southern peninsula that separates Kaipara Harbour from the Tasman Sea. Lake Ototoa and its surroundings have been described by Green (1975), Barnes and Burns (2005), Gibbs (2006), and in various publications available on the ARC web site, e.g., "Case study: Lake Ototoa" (www.arc.govt.nz/library/w45385_2.pdf, in "State of the Auckland Region Report 2004"). There it states that Lake Ototoa "has the best water quality of the lakes monitored by the ARC. The lake supports a variety of native plants within the lake and around its margin, and is home to several native fish that are uncommon in the Auckland region. Lake Ototoa is also a significant rainbow and brown trout fishery." The same publication expresses concern about a trend in declining water quality in the lake, as do conclusions presented by Barnes and Burns (2005), but more recent data has led Gibbs (2006) to question some of these conclusions.

The surrounding land use is described concisely by Gibbs (2006): "The lake lies between steep pastured hills on the eastern side and pine tree plantations on the sand dunes on the western side. The lake orientation in the resultant valley tends to direct the south-westerly – north-easterly wind flow along the axis of the lake. The lake has one stream inflow [at its northern end, see Figure 1] plus several ephemeral streams but there is no surface outlet." Further details on lake bathymetry are given in the next section.

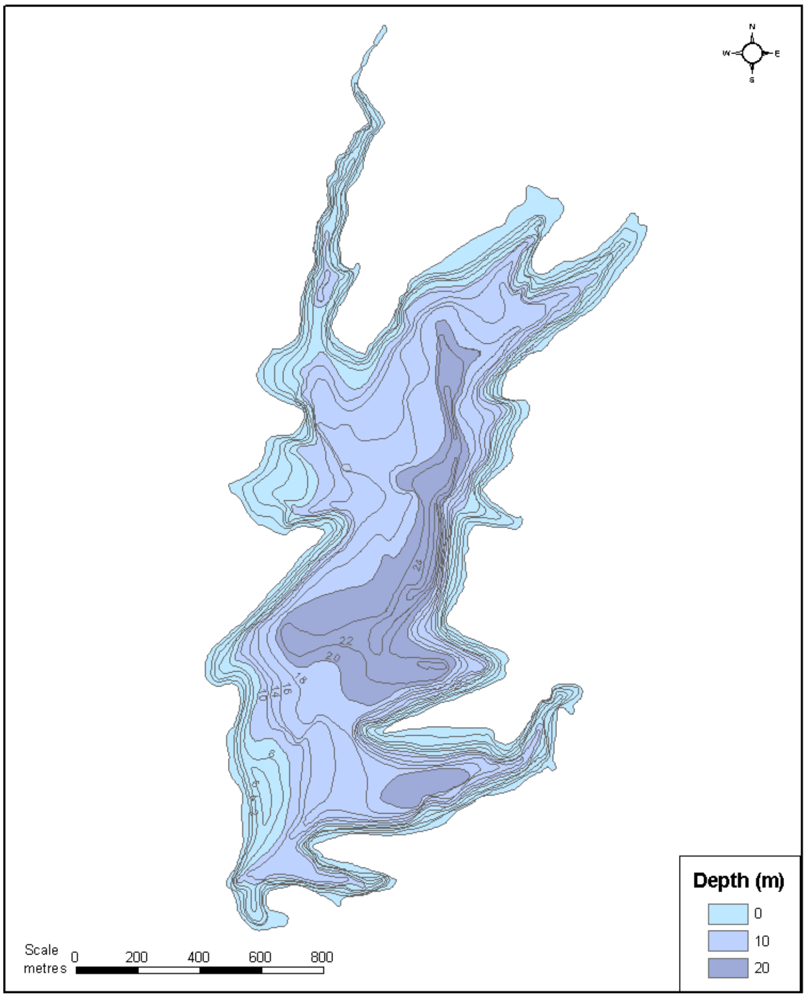
6 Input data for the model

6.1 Bathymetry

Bathymetric data used in this study are based on echo soundings made in 1970 and documented in the lake charts of Irwin (1973) and Irwin and Main (1981) (see Figure 6, redrawn from Irwin and Main 1981, and supplied by I. Hawes and M. Gibbs).

Figure 6

Bathymetric map showing depth contours of Lake Ototoa; redrawn from Irwin and Main (1981) and supplied by I. Hawes and M. Gibbs. Data are based on echo soundings made in 1970 when the lake level was 0.6 m on the gauge on the south-east arm of the lake (Figure 1).



The lake has fairly steep sides around much of its perimeter, down to about 18 m, with gentler slopes over the main part of the basin. Milder slopes also occur in the shallower parts of the lake in two northwestern bays, and there is a relatively flat shelving region at depths of 5 to 10 m that extends into the lake at the southern end. Barnes and Burns (2005, Table 4, p.9) give a maximum depth of 29 m for the lake.

DYRESM requires data for areas within depth contours and lake volumes below depth contours. Depth-area-volume data, based on planimetry of Irwin's (1973) chart, were given by Green (1975, Table 1) and are shown below in Figure 7 and Table 2 for reference. There is very little volume below 26 m, and for modeling purposes the maximum depth has been taken as 26.3 m, based on extrapolation of the depth-area curve to its intersection with the depth axis (Figure 7). Lake elevation, as given on the ARC water level gauge located on the southeast arm of the lake (Figure 1), was 0.6 m at the time of Irwin's bathymetric survey, corresponding to the level of zero depth in Figure 7 and Table 2. Daily mean water levels between 18 March 2006 – 29 April 2007 had a range between 1.023 m and 1.577 m. This period covers most of the simulation period; water level data available at the time the model runs were being set up ended on 29 April.

Figure 7

Depth-area-volume curves for Lake Ototoa, based on planimetry of depth contours in Irwin's (1973) chart by Green (1975).

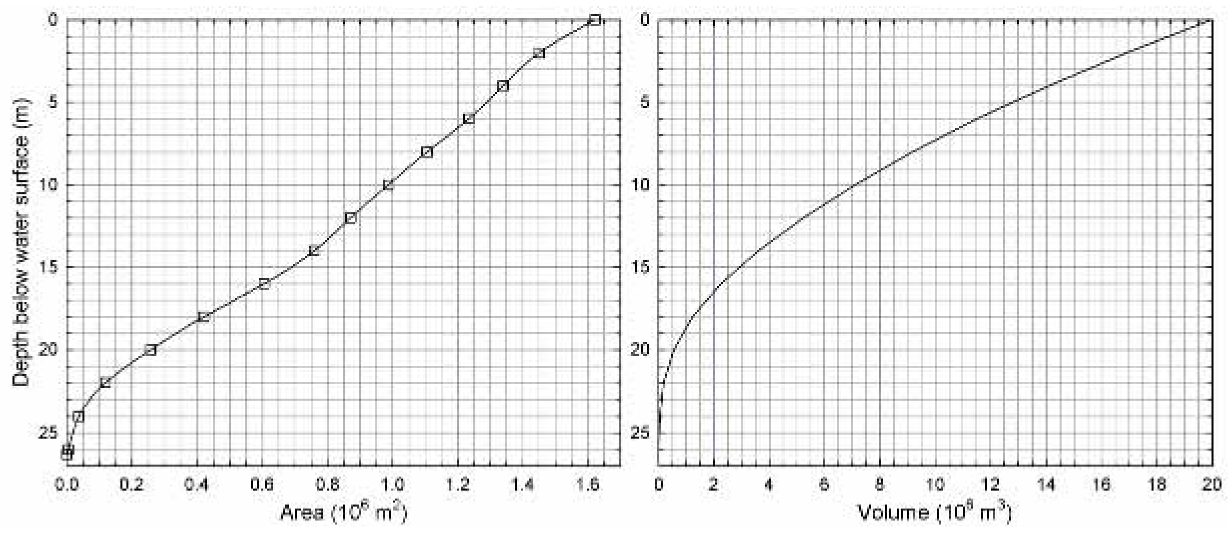


Table 2

Depth-area-volume data used for model simulations. Areas are from Green (1975). Volumes have been recomputed using the prismoidal formula (as in Green 1975) for volume V between two parallel areas A_1 and A_2 a distance h apart, $V = h/3\,[A_1 + (A_1A_2)^{1/2} + A_2]$. Results are essentially the same as in Green (1975).

Depth (m)	Area of contour (10^6 m^2)	Volume between depths (10^6 m^3)	Cumulative volume (10^6 m^3)	% total volume
0	1.623	3.0724	19.9600	100.000
2	1.451	2.7903	16.8876	84.607
4	1.340	2.5733	14.0974	70.628
6	1.234	2.3388	11.5241	57.736
8	1.106	2.0919	9.1853	46.018
10	0.987	1.8568	7.0934	35.538
12	0.871	1.6287	5.2366	26.235
14	0.759	1.3611	3.6079	18.076
16	0.605	1.0205	2.2468	11.256
18	0.421	0.6724	1.2263	6.144
20	0.258	0.3681	0.5540	2.775
22	0.119	0.1482	0.1858	0.931
24	0.037	0.0371	0.0376	0.188
26	0.005	0.0005	0.0005	0.003
26.3	0.000		0.0000	0.000

6.2 Meteorological data

Meteorological data are used in the model to compute exchange of heat, water vapour and momentum between the lake and the atmosphere. In a lake like Ototoa, where stream inflows are relatively small and there are no geothermal inputs, these exchanges dominate the thermal regime of the lake.

The daily meteorological data with gaps filled in are shown in Figure 8. A few comments regarding the data can be made at this point; further references will be made later in the discussion of model results.

In addition to solar radiation measured at the weather station, solar radiation outside the atmosphere was computed from formulas found in meteorology texts, e.g., Monteith (1973). The solid red curve designated “solar radiation under clear skies” uses this calculated quantity in an empirical relation from Monteith (1973, p. 28) to estimate maximum possible solar radiation that might be expected at the lake; it is used as a quality control check on the measured values. The pink and green curves in the solar radiation graph provide approximate upper and lower envelopes for the measured data, and were used to estimate mean daily values of cloud cover. Cloud cover is used in the model to calculate longwave (infrared) radiation from the atmosphere (TVA 1972). Measured values of incoming longwave radiation can be specified as part of the model input, but this is not a routine meteorological measurement and sensors are temperamental, so provision is made in the model for calculating it from cloud cover and air temperature.

Vapour pressure in the air (computed from relative humidity and air temperature) is used in the model to compute evaporation from the lake surface and the heat loss that accompanies evaporation, called *latent heat flux*. For every gram of water evaporated, approximately 2500 joules of heat, the latent heat of vaporization, is lost from the water surface. Air temperature is also used in the calculation of *sensible heat flux*, which is heat lost (or gained) because of any difference in temperature between the water surface and the air above it. Both sensible and latent heat fluxes rely on wind and turbulence in the air to carry heat and water vapour away from the lake surface. The calculations for sensible and latent heat fluxes assume that the fluxes are directly proportional to wind speed.

In addition to assisting in transport of heat and water vapour between the lake and the atmosphere, the wind also imparts momentum to the water surface, thereby generating waves, currents and turbulence. Wind thus plays a crucial role in mixing heat downward in the water column. Although DYRESM, being a one-dimensional model, does not model wind-generated lake currents, it does account for the energy imparted by the wind to the water surface in its calculations for turbulent mixing.

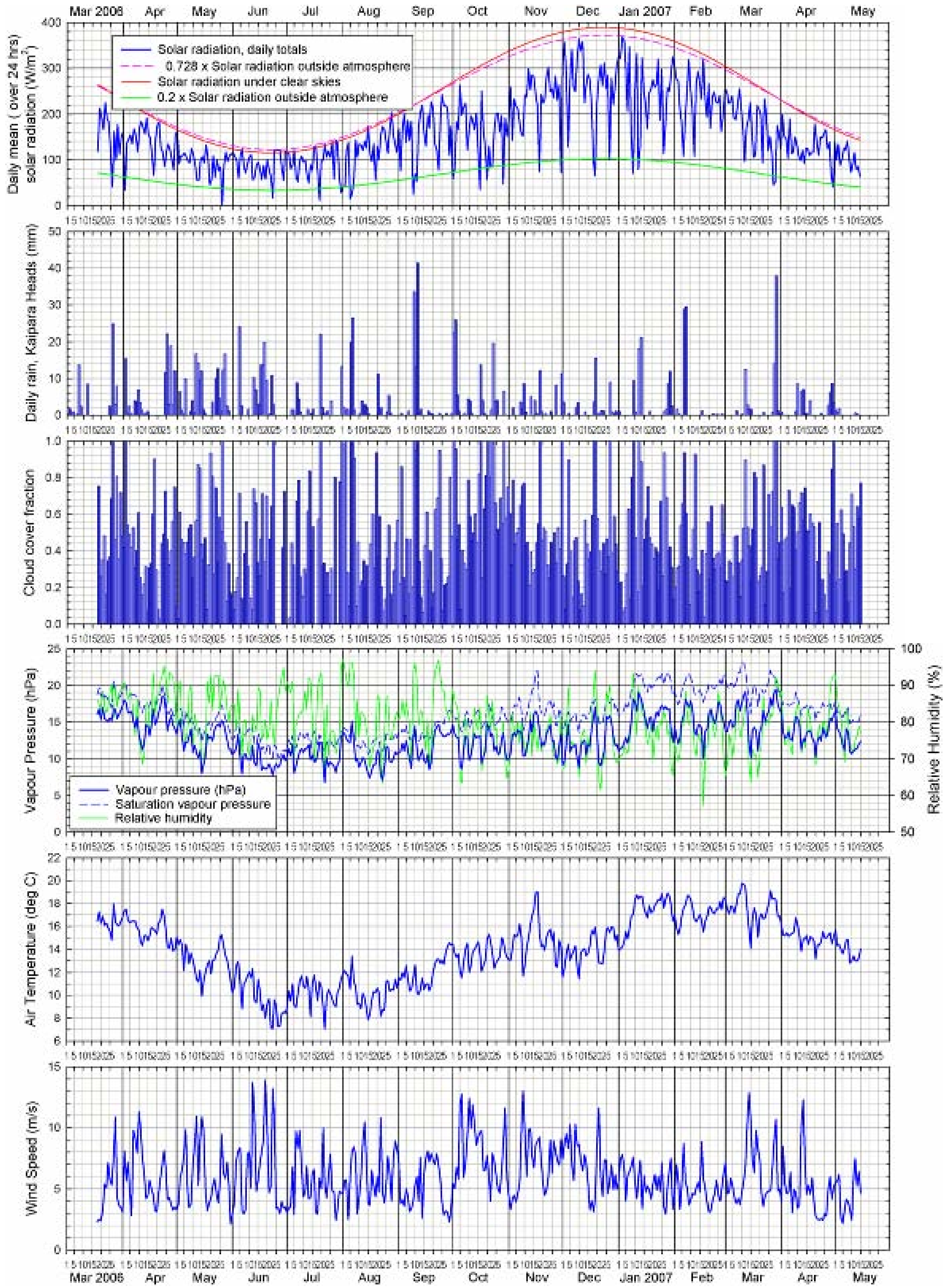
Because the weather station is located on a hill 90-100 m above the lake surface, there are some differences that can be expected between air temperatures and wind speeds measured at the weather station versus those that occur immediately over the lake. In a neutrally stable dry atmosphere air temperatures decrease with elevation by 9.8°C km^{-1} (the dry adiabatic lapse rate; Monteith 1973). In an atmosphere at 100% humidity, temperatures only increase at about half that rate (4.9°C km^{-1}, the saturated adiabatic lapse rate) because of condensation and the associated release of the latent of condensation. The actual decrease will generally be somewhere between these two extremes (except under conditions of strong atmospheric stability or instability, conditions that are usually weakened by wind). In the model application, air temperatures measured at the weather station were increased uniformly by 0.5°C, corresponding to a lapse rate of 5°C km^{-1} over the roughly 100 m difference in elevation between the weather station and the water surface.

Wind speed can be expected to increase with height in a neutrally stable atmosphere. It also can be expected to increase over the crest of hills. There is no simple way to compute the expected increase due to these two factors combined for a complex natural terrain. However, it was felt that some adjustment needed to be made to measured wind speeds, which can be seen in Figure 8 to be generally quite high, especially for daily means. The correction that was made was to reduce mean daily wind speeds measured at the Te Rokotai weather station by a factor of 1.4. This is based on the regression equation for daily mean wind speed measured at Ototoa (U_{Ototoa}) versus daily mean wind speed at Dargaville ($U_{Dargaville}$), U_{Ototoa}[m s^{-1}] = 0.52 + 1.37 $U_{Dargaville}$[m s^{-1}] (Section 2).

No changes were made to solar radiation, rainfall or relative humidity. The correction for air temperature was incorporated in the calculation of vapour pressure.

Figure 8

Meteorological data (all daily means) used for modeling



6.3 Inflows and outflows

DYRESM requires data for daily volumes and temperatures (and salinities if they are significant) of all major inflowing streams. It also requires data for daily outflow volumes. If outflows are not specified, the model incorporates them into any overflows that are calculated as part of the daily water balance. The net heat flux supplied to the lake by streams, due to differences in temperature between stream inflows, outflows and the lake, is referred to as an *advective heat flux*. The term implies that the heat is transported or "advected" by the flow of water.

The only perennial stream of consequence for the model application enters the lake at its northern end (Figure 1; photo Figure 9). It is not permanently gauged and baseflows are small; spot gaugings by Gibbs (2006) were in the range 10 – 20 L s^{-1}.

To estimate inflows and outflows for the purposes of the model application, the daily meteorological data shown in Figure 8, together with daily mean water levels (calculated from 15-minute data supplied by ARC), were used to calculate a lake water balance as:

$$\Delta Z / \Delta t = (Q_{in} - Q_{out}) / A + P - E \qquad (1)$$

where Z/ t is rate of change in lake level, A is lake surface area (1.623 km^2, Table 2), P and E are rainfall on and evaporation from the lake surface, and Q_{in} and Q_{out} include all surface and groundwater inflows and outflows. Z/ t, P and E can be evaluated from meteorological and water level data, and are therefore known. The net inflow (inflow minus outflow) can be calculated using Equation 1. All terms in the above equation are illustrated in Figure 10.

Evaporation from the lake surface was calculated for the water balance as (see, e.g., Monteith 1973, TVA 1972):

$$E = (\rho_{air} / \rho_{water})\, C_E\, U\, (q_L - q_{air}) \qquad (2)$$

where $_{air}$, $_{water}$ are densities of moist air and liquid water, C_E is a mass transfer coefficient (assumed to be 1.0×10^{-3}), U is wind speed (the measured wind speed divided by 1.4, as described above), and q_L, q_{air} are specific humidities at the lake surface and in the air. q_L was calculated as the saturation specific humidity corresponding to the lake temperature measured by the ARC thermistor at 1 m depth. All values of meteorological parameters were daily means for the water balance calculations of evaporation. C_E, q_L and q_{air} are dimensionless, so E in Equation 2 will have the same units as U. This is essentially the same method used to calculate evaporation in the model, except the model calculations are made at hourly time steps and use predicted lake surface temperatures. This will change the values for q_L and hence for E, so the estimate of evaporation calculated for the water balance cannot be expected to be exactly the same as that calculated in the model simulations.

Figure 9

Stream inflow; ARC photo.

Figure 10

Water balance terms; inflow – outflow (Q_{in} – Q_{out}) calculated from Equation 1

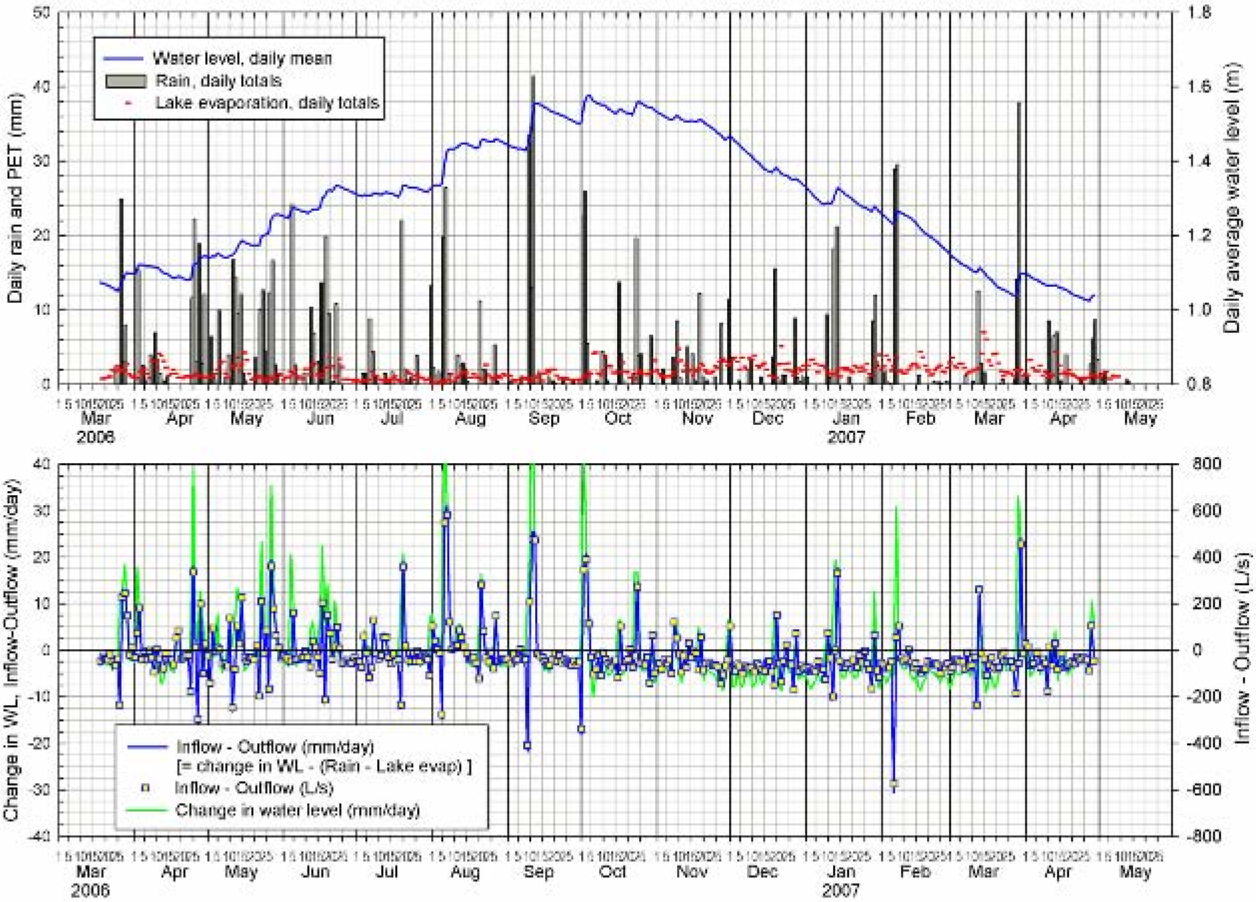


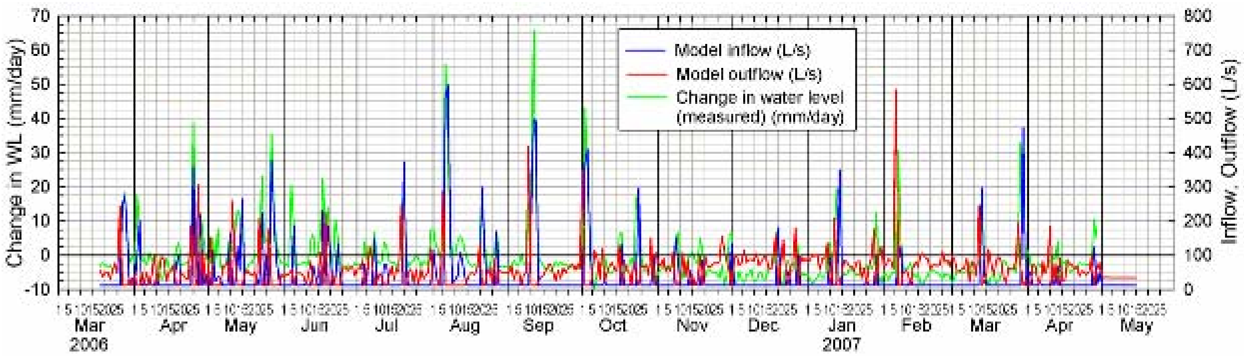
Figure 10 shows that most of the time outflows exceeded inflows; the average value of *I-O* (= $Q_{in} - Q_{out}$) over the period 18 March 2006 – 29 April 2007 was -1020 m^3 day^{-1} (-11.8 L s^{-1}, or -0.629 mm day^{-1} on a per unit lake surface area basis). Average rainfall over this period was 2.806 mm day^{-1}, average calculated evaporation was 2.271 mm day^{-1}, and average rate of change in lake level was -0.0933 mm day^{-1}

Two difficulties remain. First, Equation 1 only gives net inflow (*I-O* = $Q_{in} - Q_{out}$), whereas DYRESM needs times series for inflows and outflows separately. Secondly, some assumption also needs to be made regarding how to treat groundwater versus surface inflows. A third question is how to estimate inflow temperatures.

Inflow volumes were estimated as follows. On days when *I-O* was negative (i.e., outflows exceeded inflows), inflow volumes were assumed to be at a baseflow level of 1000 m^3 day^{-1} (11.6 L s^{-1}). On days when inflows exceeded outflows (*I-O* > 0, generally during storms), inflow was taken as *I-O* + 1000 m^3 day^{-1}. To close the water balance outflow was calculated as inflow minus *I-O* [i.e., $Q_{out} = Q_{in} - (Q_{in} - Q_{out})$]. Because of the discrepancies between evaporation calculated for the water balance and evaporation calculated by the model during the simulations, it was found necessary to slightly increase the estimated outflows for the model simulations to get a good match between measured and modeled lake surface levels (Figure 11); this had no effect on modeled temperatures. Following discussion with Max Gibbs, the outflow was assumed to leave the lake via seepage at a depth of 10 m. Values of inflows and outflows used for the model simulation are shown in Figure 11.

Figure 11

Model inflows and outflows (right-hand axis), and change in measured daily mean water level (left-hand axis).



All inflow was treated as coming from a single source for the purposes of the model application. No attempt was made to separate out the contribution from groundwater and treat this as a separate input. Little is known about the amount and distribution of groundwater flows, but they are not likely to be very large considering the size of the catchment, the lake's elevation and its location in the middle of a narrow peninsula. It is assumed that most of the groundwater inflows are likely to be focused in the littoral zone and have similar temperatures and effects on the lake's thermal regime as the inflow stream.

The final specification required for inflows is a time series of daily mean stream temperatures; salinity effects were assumed negligible. No information was available for stream temperatures. Following Mosley (1983), mean daily stream temperatures were assumed to be described by a single sine curve of the form $T(t) = T_{avg} + T_{amp}$ sin[2 (t +)/365], where t is day number in the year, T_{avg} is annual mean temperature, T_{amp} is amplitude of the temperature variation about the mean, and is a phase shift. Values of T_{avg} = 17°C, T_{amp} = 5°C and = 65 days were used in the model; these values are averages of the parameter values given in Mosley (1983) for the three closest sites to Lake Ototoa that are shown in his study. This variation gives maximum stream temperatures of 22°C at the end of January and minimum temperatures of 12°C at the end of June.

Although all of the above assumptions made for volumes and temperatures of stream inflows seem reasonable, inflow-outflow dynamics remain an unknown factor.

Fortunately for the model application, as will be seen in the discussion of results, inflows and outflows appear to play a minor role in the lake's heat balance.

6.4 Light attenuation coefficient

The model requires a user-specified value for the attenuation coefficient, K_d, for downwelling irradiance in the water column. Irradiance is the downward component of solar energy flux per unit horizontal area, with units usually given as watts m^{-2}. K_d is used in the model to determine how the incoming solar radiation incident at the lake surface is distributed with depth in the lake. The dimensions of K_d are the inverse of length (1/length), with units usually of 1/metres (m^{-1}). Typical values of K_d range from less than 0.1 m^{-1} in very clear oligotrophic lakes, to greater than 3 m^{-1} in turbid, deeply coloured lakes (see, e.g., Davies-Colley et al 1993).

K_d plays an important role in determining the depth of thermoclines and the strength of temperature stratification in lakes. Lakes with higher values of K_d trap solar radiation at shallower depths, leading to warmer surface layers and cooler water at depth. In clear lakes, with smaller values of K_d, solar radiation penetrates to deeper depths, thereby spreading the solar heat over greater volumes of water, leading to deeper thermoclines.

As far as we are aware, no direct measurements have ever been made of irradiance or K_d in Lake Ototoa. Green (1975) measured profiles of illuminance and derived attenuation coefficients for illuminance. Illuminance is the visual counterpart of irradiance; it is generally measured with a photometer using a filter that matches the spectral response of the human eye, and is usually specified in units of lumens m^{-2} (= lux). Because of the dependence of illuminance on the spectral composition of light, there is no way to compute irradiance from total illuminance values. Nevertheless, it seems reasonable to assume that attenuation coefficients for downwelling illuminance should provide a good approximation to attenuation coefficients for downwelling irradiance. Green's (1975) illuminance attenuation coefficients varied from 0.15 m^{-1} to 0.5 m^{-1}. This wide range is matched by the range in clarity as measured by Secchi disk depth measurements for the reasons described below.

Green (1975) and Gibbs (2006) both present values of Secchi disk depth (Z_{SD}). Secchi disk depth is often used synonymously with water clarity and is a measure an observer's ability to perceive visual contrast in a water body. Green's (1975) measurements of Z_{SD} varied from 5 m to 9 m around a mean of approximately 7 m. Later measurements made by ARC, starting in 1990 and presented in Gibbs (2006), vary from less than 2 m to approximately 8.5 m, with a mean of 4.7 m. Both Green (1975) and Gibbs (2006) comment on the high degree of variability of Z_{SD} in Lake Ototoa and attribute it to variations in phytoplankton populations (both type and density) and suspended mineral particles blown into the lake from neighbouring sand dunes.

Although Z_{SD} is related to K_d, it does not measure the same thing that K_d measures, and there is no unique relation between Z_{SD} and K_d. An inverse correlation between Z_{SD}

and K_d that is frequently quoted is that of Pool and Atkins (1929) (see Davies-Colley et al., p. 77):

$$Z_{SD} = 1.7 / K_d \quad (3)$$

The values of K_d corresponding to the minimum, mean and maximum values of Z_{SD} of Green (1975) as given by this relation are 0.34 m^{-1}, 0.24 m^{-1} and 0.19 m^{-1}; the corresponding values of ARC and Gibbs (2006) are 0.85 m^{-1}, 0.36 m^{-1} and 0.20 m^{-1}.

On the basis of the illuminance attenuation coefficients and Secchi disk depth correlations, as discussed above, a value of K_d = 0.30 m^{-1} was chosen for the model simulations. This single value was used for the entire simulation period. At present there is no facility for including variability of K_d in DYRESM simulations.

6.5 Other model parameters

There are three further coefficients that enter the calculations in DYRESM for turbulent mixing in the epilimnion. These arise in parameterizations of terms in the turbulent kinetic energy budget and suggested values are based on experiments described in the scientific literature on turbulence, completely independent of DYRESM, as described by Imberger (1979), Imberger and Patterson (1981, 1990) and more recently by Yeates and Imberger (2003) (also see on-line documentation for DYRESM as described earlier). A further parameter controls weaker diffusive mixing in the hypolimnion (Yeates and Imberger 2003). A recently-added benthic boundary layer mixing routine (Yeates and Imberger 2003) was not invoked in these simulations. There is some scope for varying the above parameters to achieve better agreement between simulated and measured temperatures, but is it is limited.

7 Results and comparison with measured temperatures

Results of the simulations are presented below in three ways: as predicted temperature contours in the time-depth plane, compared with a similar plot for temperatures measured by the ARC thermistor chain; as time series of predicted temperatures at the mooring depths, compared with measured time series at the thermistor mooring; and as profiles on days when ARC measured temperature profiles in the lake as part of their routine monitoring programme. In Section 7.1 the overall features of the seasonal cycle, as measured and as modeled, will be summarized and discrepancies between model predictions and measured temperatures are discussed. In Sections 7.2 and 7.3 some aspects related to lake water balance and heat balance are discussed.

7.1 Overall comparison and the seasonal cycle

Temperature contours (Figure 12) and time series (Figure 13) give an overall impression of the successes and failures of the simulations. The model accurately captures the overall patterns of stratification and mixing, in terms of timing, strength of stratification and depth of the thermocline. The agreement on precise temperatures within the simulation is not exact, however, which is to be expected; discrepancies will be discussed below. A brief description is given first of the seasonal cycle, with reference to Figures 12, 13 and 14.

Model simulations begin in late summer on 18 March 2006, the starting date for complete daily meteorological data, and end in late autumn on 8 May 2007 (Figure 15). The lake is still stratified when simulations begin, with a relatively deep thermocline. Following a brief warming period in the first 2 days of the simulation, the lake begins to cool and the thermocline deepens. The lake is vertically mixed by June, and continues to cool until the start of July. The model predicts further cooling, albeit at a much slower rate, through July, but the measured temperatures remain fairly constant. Warming begins slowly in August, accompanied by very weak stratification. Warming continues at a more rapid pace in September, and stratification gains strength thereafter.

The warming is temporarily interrupted at the beginning of October by a period of cooler air temperatures and strong winds, but warming resumes in mid October (also see sensible and latent heat flux terms in the top graph of Figure 16, which will be discussed in Section 7.2). Steps can be seen in the time series for temperatures below 4 m at various times throughout the years (Figure 13). These are the response to deepening of the thermocline caused by turbulent mixing that is generated by strong winds and/or surface cooling. The step in temperature occurs as warm water from a thickening epilimnion mixes with the colder water at the level of a thermistor. A set of these steps accompanies the windy period at the start of October in the graphs for both modeled and measured temperatures in Figure 13.

Figure 12

Contours of constant temperature in the time-depth plane as measured by ARC thermistor chain (top graph) and as predicted by the model (bottom graph).

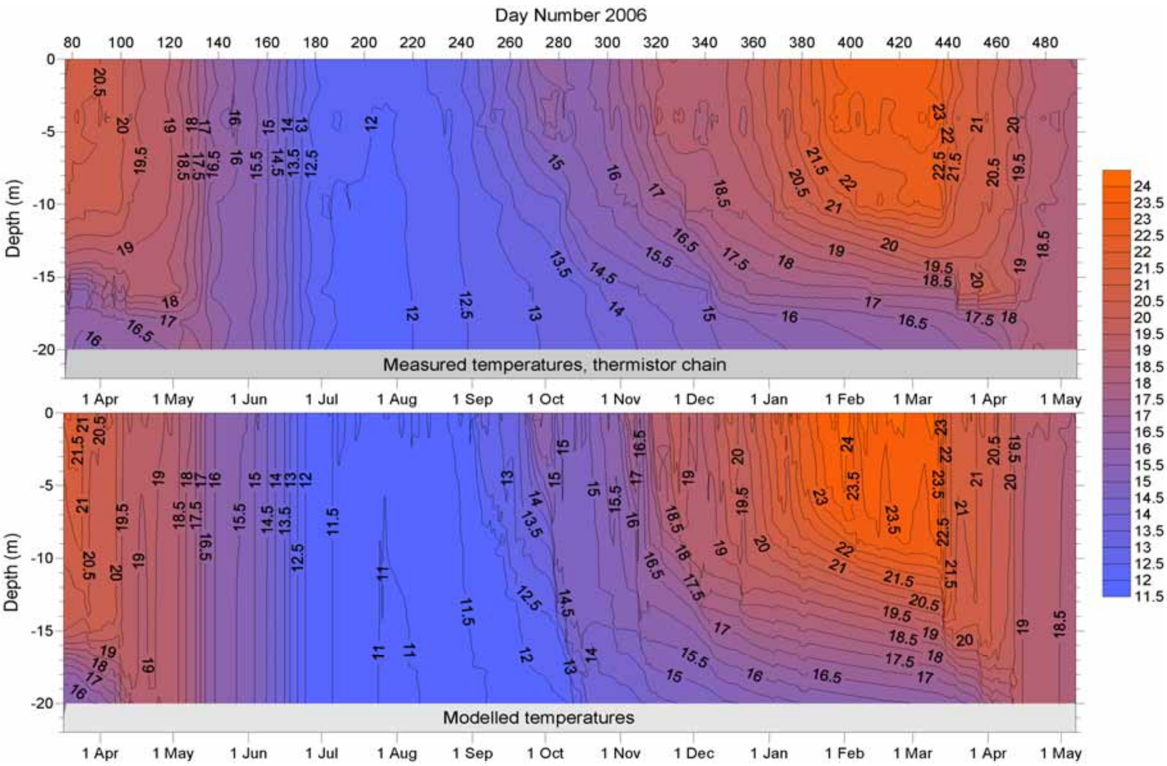


Figure 13

Time series of temperatures measured by ARC thermistor chain (daily means, top graph) and as predicted by the model (output at 14:00h, bottom graph).

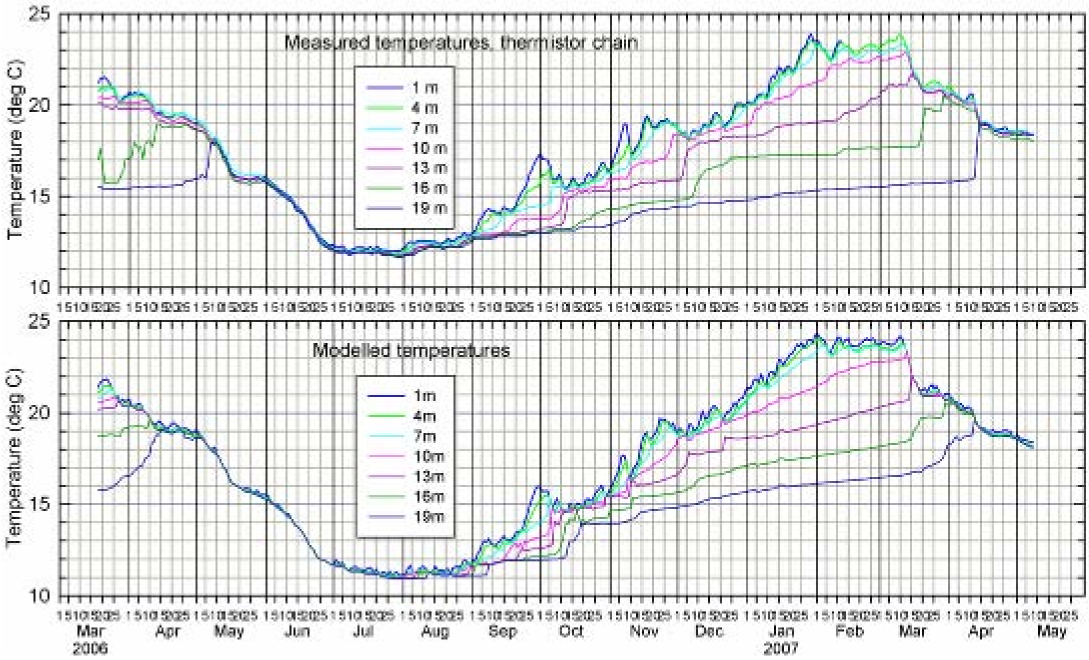
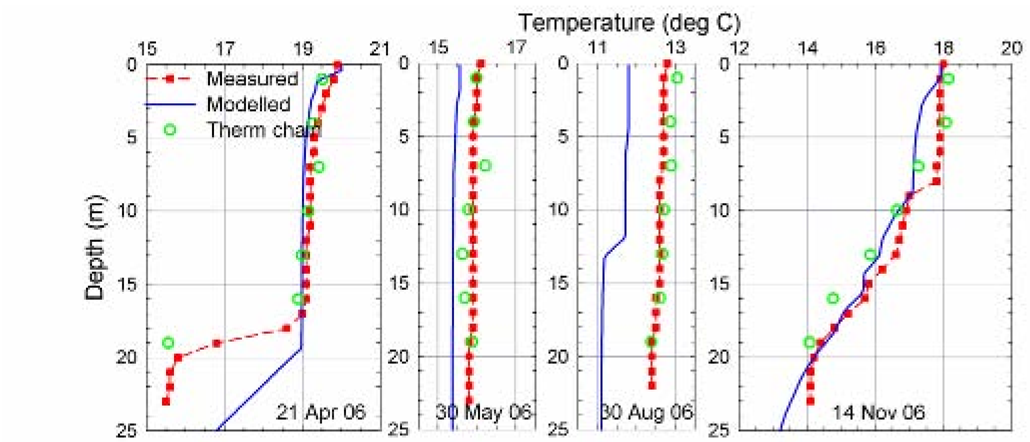
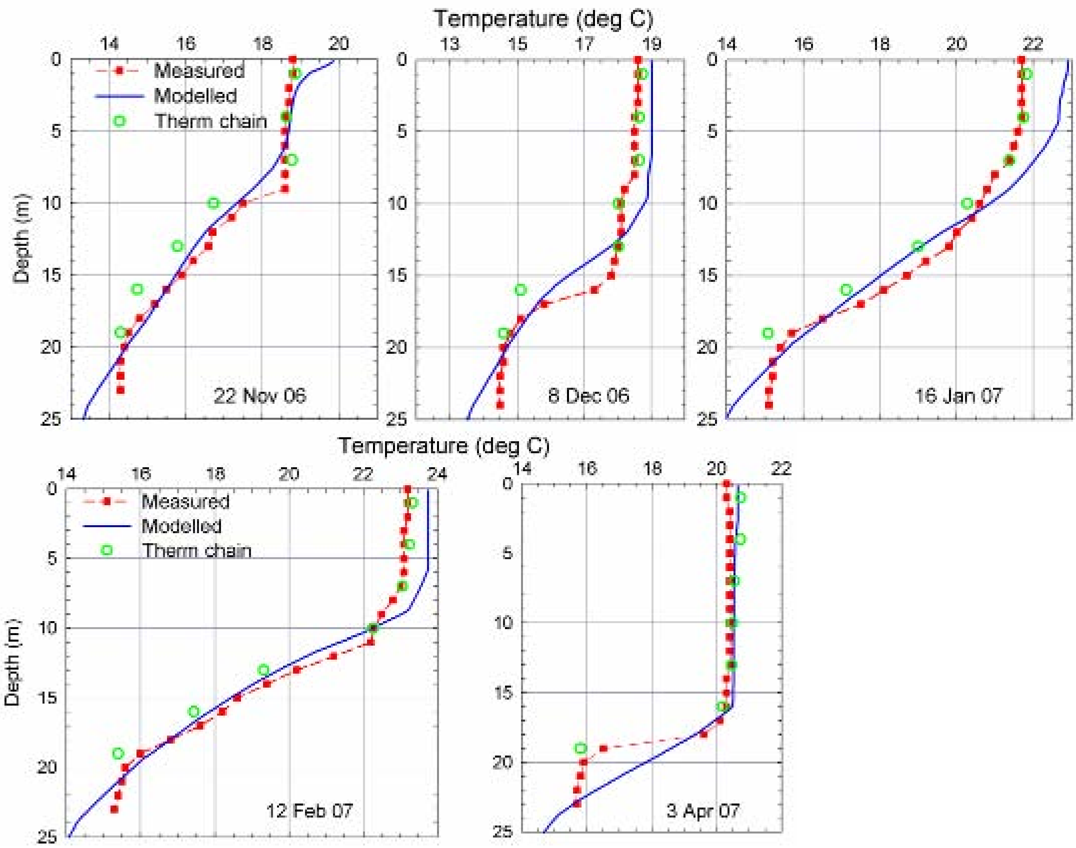


Figure 14

Temperature profiles measured by ARC during their routine monitoring visits (red squares) compared with model predictions on those days (blue line). Temperatures measured by the ARC thermistors at the times of profile measurements are also shown (green circles).





As noted by Green (1975), the pattern of surface temperatures tend to follow the pattern of air temperatures, remaining near maximum levels from late January through early March, when the lake attains its maximum heat content (Figure 12). Cooling after 10 March 2007 is rapid, and vertical mixing occurs by the end of the simulation (8 May 2007). Comparison of predicted maximum, mean and minimum temperatures with those measured by the thermistors for the entire simulation period are shown in Table 2. The agreement is very good, with discrepancies being generally within, or close to, the bounds of accuracy of the Onset thermistors.

Evolution of the thermocline can also be observed in the profiles of Figure 14. The lake does not seem to develop the classic textbook three-layer stratification of epilmnion-thermocline-hypolimnion. Epilimnion thickness is variable, and the thermocline is broad, containing considerable fine-structure, perhaps better described as consisting of multiple thermoclines. Gibbs (2006, pp. 9-10), referring to temperature profiles (not shown here) measured on 5 December 2005, 12 January 2006 and 20 March 2006 observed that “the thermal structure of the lake is complex with a double thermocline, the first around 6 m and the second at around 15 m. This double thermocline was also present in the 12 January and 20 March profile data indicating that it is a feature of the lake rather than an unusual on-off observation”. Green (1975, p. 204) observed that “a feature of the [temperature profiles] is the small irregularities in the epilimnion on many occasions … These microstratifications probably result from surface heating and subsequent incomplete mixing by the wind.” It is common for lakes to develop shallow diurnal thermoclines on calm days, with a deeper and stronger seasonal thermocline below (e.g., Imberger and Patterson 1990). Multiple deeper thermoclines can represent remains of past wind or surface cooling induced mixing events that have since subsided. This is similar to Green's (1975) hypothesis, and is how the structures observed in the model profiles were generated. Other processes that can lead to multiple thermoclines are intrusions from stream or groundwater inflows; intrusions of mixed fluid generated in the benthic boundary layer along the bed of the lake by wind-induced seiching (both internal and external modes); and compression of vertical thermal structure by persistent submerged withdrawal of fluid from depth. Of these, the second seems the most applicable to Lake Ototoa in view of the strong winds and consequent seiching that occur there (see also comments by Gibbs 2006 regarding tilting of the thermocline under wind). Inflows and outflows, at least as estimated here, are too small to have a major influence on thermal structure.

There are two notable discrepancies between measured and modeled profiles. The first is the offset of approximately 1°C between measured and modeled profiles on 30 August – 1 September 2006. This is associated with the weak cooling predicted by the model for July that is not observed in the measured temperature data. The cooling predicted by the model is a straightforward result of the heat flux calculations carried out in the model. In Figure 16, the top graph shows that calculated heat losses due to sensible and latent heat fluxes exceeded gains from net radiation in July, resulting in a net heat loss from the lake to the atmosphere. The problem could be due to the heat flux calculations (which are fairly standard, but are based on daily mean data), the meteorological data, or a combination of the two. The second discrepancy is with the failure of the model to predict a nearly mixed layer near the bottom once stratification develops. After September, the blue curves in Figure 14 tend to continue their trend of smooth temperature decrease predicted for the thermocline all the way to the bottom. The measured profiles indicate the existence of a nearly mixed layer in the bottom 5 m of the lake that may be due to mixing in the boundary layer at the lake bed, caused by internal seiching. Water volumes below 20 m are small, the bed is fairly flat, and the lake is subject to frequent strong winds – ideal conditions for bottom boundary layer mixing. DYRESM has been modified in the past to account for this process in an application to Lake Erie (Ivey and Patterson 1984) but the present version does not incorporate this modification.

7.2 Water levels

A comparison between modeled and measured daily water levels over the period of the simulation is shown in Figure 15. This provides a check on the overall water balance calculations that accompany the heat balance and mixing calculations in the model. The agreement is not perfect, but it is reasonably good, and the small discrepancies have no direct impact on the heat balance and mixing calculations. Rather, they may prove of assistance when trying to diagnose possible weaknesses in the evaporation calculation or the estimates for inflows and outflows. In this case it is difficult to know exactly how to interpret the relatively small discrepancies between modeled and measured water levels, given the uncertainties associated with the inflow and outflow volumes and with the meteorological data.

Figure 15

Model predictions of daily water levels (green line) compared with measured values (blue line) from the ARC gauge. Measured-modelled differences are shown by the black line; note the different scale on the right-hand-side of the graph.

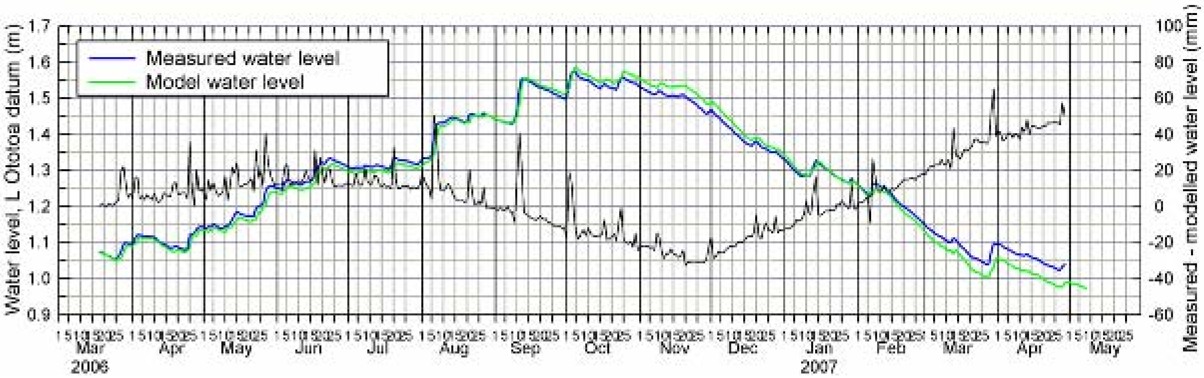
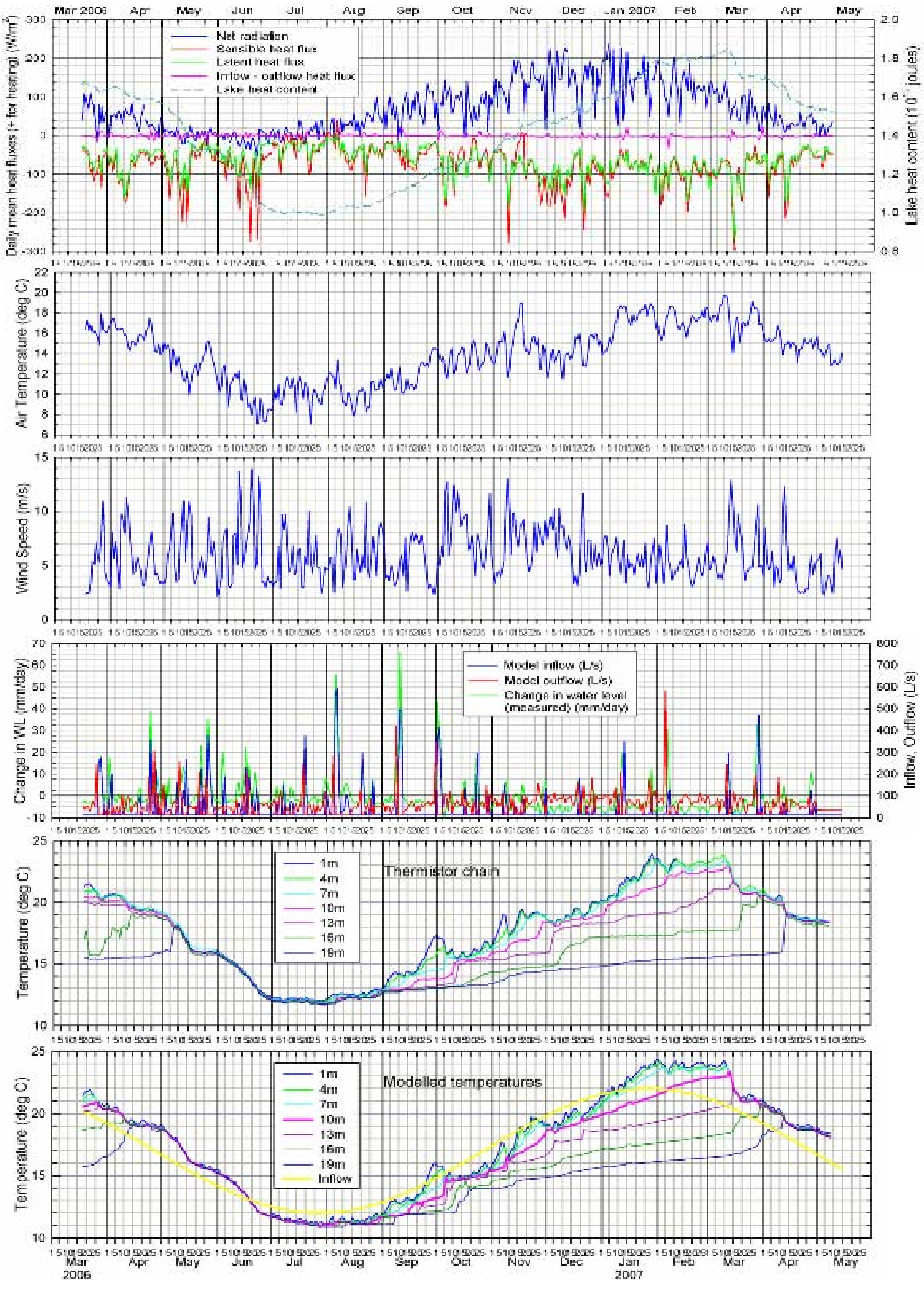


Figure 16

Lake heat balance terms (top graph), with accompanying graphs.



7.3 Heat fluxes

In order to get some idea of the relative importance of the various factors that influence the thermal regime and temperature structure of the lake, heat fluxes calculated from the meteorological data and surface water temperatures (as measured at the 1 m depth ARC thermistor) are shown in Figure 16 (top graph). The fluxes have been calculated using standard formulae given, for example, in TVA (1972), Monteith (1973) and Imberger(1979). Other graphs in Figure 16 repeat plots given earlier, but are put here for reference to assist in interpreting the causes and effects of the fluxes relative to lake temperatures and mixing. All fluxes are daily means over 24 hours.

The major source of heat for the lake is net radiation at the water surface (blue curve, top panel, Figure 16), consisting of: incoming solar radiation minus reflected solar radiation, plus incoming longwave radiation from the atmosphere minus reflected atmospheric longwave radiation, minus outgoing longwave (almost-black-body) radiation from the water surface. Net radiation is generally positive except for periods in winter.

The major heat losses are latent (green) and sensible (red) heat fluxes associated with evaporation, wind and air temperatures that are cooler than the water surface. Occasionally these fluxes are positive, resulting in heat gain, if there is net condensation instead of evaporation, and if air temperatures are warmer than the water surface. The graphs of wind and air temperature help explain the variations in these fluxes. For example, the very large values of sensible and latent heat losses as three spikes in June are clearly associated with concurrent large values of wind speed.

The net advected flux due to inflows and outflows (pink curve) is basically the difference between the product of inflow discharge and inflow temperature, minus the product of outflow discharge and outflow temperatures (multiplied by water density and specific heat capacity). One of the most striking aspects of the graph is how small this contribution is compared to the other fluxes. The components of the flux are also plotted in Figure 12 for reference – inflow and outflow discharges (fourth graph), and inflow and outflow temperatures (yellow and pink lines in the bottom graph; note that outflow temperature is assumed to be identical to lake temperature at 10 m depth). Most of the time the advective flux is negative, resulting in a small net heat loss. This is a result of the outflows exceeding inflows, on average.

The heat flux graph illustrates very clearly the relative importance of heat and mass transfer between the water surface and the atmosphere, compared with advective fluxes, in the overall heat balance of the lake.

The total heat content of the lake (computed relative to zero heat at 0°C) is also shown in the top graph (dashed curve, axis on right-hand-side). It lags behind net radiation, but appears to be generally in phase with air temperature. Heat content is minimum at the end of July and maximum in early March.

Finally, Figure 16 helps to explain some of the features observed in the graphs for lake water temperatures (bottom 2 graphs, repeated from Figure 13). For example, the variations in cooling rate in May and June, with steeper slopes of the temperature curves in the first half of May and the middle to late part of June, are clearly the result of corresponding large sensible and latent heat during those periods.

8 Scope for longer-term simulations and water quality simulations

The quality of the simulations is good enough to warrant the conclusion that the model could be used to model effects of climate change or as a base for coupled hydrodynamic – water quality modelling. The limiting factor in either case is the availability of input data to run the model. For climate change investigations, a long-term time series of meteorological data, that spans the period over which climate change is being considered, would need to be assembled. To model water quality, data on nutrient fluxes (including various forms of nitrogen, phosphorus and carbon – dissolved vs. particulate, organic vs. inorganic, labile vs. refractory) would need to be available in sufficient detail to permit daily time series for loadings to be assembled. In-lake data on nutrients, oxygen and phytoplankton would also be required to check model predictions and to calibrate model constants and coefficients. Unlike the coefficients for the physics-based modelling, water quality model coefficients are numerous and do require careful adjustment.

Longer term meteorological data are available from Dargaville, as summarized in Table 3. Given the regression equations that have been developed for the present study and that are set out Section 3, these could be converted to an Ototoa meteorology dataset for longer-term simulations. It needs to be kept in mind, however, that data predating the electronic weather station (EWS) may not be available at a high enough time resolution to permit truly reliable estimates of daily means. In addition some way would have to be found to fill in gaps in data listed in Table 3, and quality of the data would have to be checked. Past experience with assembling a long term (25-year) meteorology data set for Lake Taupo by piecing together the best possible data from a variety of sources has shown that it is not a trivial exercise (Spigel et al. 2001, 2003). Finally, it would be necessary to agree on a meaningful climate change scenario (or scenarios) for testing.

Table 3

Availability of long-term meteorological data

Site No	Site name	Parameter	Start time	Finish time
539806	Dargaville EWS	Solar radiation	24 Nov 1997 16:00	1 Jun 2002 10:00
539807	Dargaville2 EWS	Solar radiation	29 Oct 2003 15:00	(Ongoing)
539802	Dargaville	Sunshine hours	1 Jan 1972 23:59	31 Dec 1998 22:59
539807	Dargaville2 EWS	Sunshine hours	29 Oct 2003 15:00	(Ongoing)
539802	Dargaville	Temps/humidity	1 Jan 1972 09:00	31 Aug 1999 09:00
539803	Dargaville NZED	Temps/humidity	28 Jul 1978 12:00	13 Sep 1988 15:00
539806	Dargaville EWS	Temps/humidity	2 Oct 1997 09:00	1 Jun 2002 09:00
539807	Dargaville2 EWS	Temps/humidity	29 Oct 2003 15:00	(Ongoing)
539802	Dargaville	Max & min temps	18 Oct 1951 09:00	31 Aug 1999 09:00
539803	Dargaville NZED	Max & min temps	13 Jan 1982 08:00	9 Aug 1988 09:00
539806	Dargaville EWS	Max & min temps	21 Oct 1997 08:00	1 Jun 2002 09:00
539807	Dargaville2 EWS	Max & min temps	29 Oct 2003 15:00	(Ongoing)
539802	Dargaville	Wind speed/dir	1 Jan 1972 09:00	31 Aug 1999 09:00
539803	Dargaville NZED	Wind speed/dir	28 Jul 1978 12:00	13 Sep 1988 15:00
539806	Dargaville EWS	Wind speed/dir	21 Oct 1997 18:00	1 Jun 2002 10:00
539807	Dargaville2 EWS	Wind speed/dir	29 Oct 2003 15:00	(Ongoing)
539802	Dargaville	Barometric press	8 Oct 1990 14:00	31 Aug 1999 09:00
539803	Dargaville NZED	Barometric press	28 Jul 1978 12:00	13 Sep 1988 15:00
539807	Dargaville2 EWS	Barometric press	29 Oct 2003 15:00	(Ongoing)
539802	Dargaville	Penman PET	7 Jan 1981 08:00	1 Apr 1988 09:00
539806	Dargaville EWS	Penman PET	26 Nov 1997 08:00	1 Jun 2002 09:00
539807	Dargaville2 EWS	Penman PET	1 Nov 2003 08:00	(Ongoing)
539802	Dargaville	Rainfall	3 Jan 1943 09:00	1 Sep 1999 09:00
539803	Dargaville NZED	Rainfall	3 Jan 1965 09:00	9 Aug 1988 09:00
539806	Dargaville EWS	Rainfall	1 Oct 1997 18:00	1 Jun 2002 10:00
539807	Dargaville2 EWS	Rainfall	29 Oct 2003 15:00	(Ongoing)
644211	Kaipara Heads	Rainfall	5 Mar 1999 14:43	(Ongoing)

9 Conclusions

The model appears to have done a creditable job in simulating the thermal regime of Lake Ototoa over a season. When compared with measured temperatures, it has reproduced the overall patterns of stratification and mixing, in terms of timing, strength of stratification and depth of the thermocline. In addition, values for maximum, minimum and mean temperatures predicted at the moored thermistor depths agreed well with temperatures that were measured there.

The quality of the simulations seem to be good enough for the model to be used to model effects of climate change or as a base for coupled hydrodynamic – water quality modeling. The limiting factor in either case is the availability of input data to run the model.

A number of assumptions had to be made with regard to meteorological data and inflows in order to assemble the input data required to run the model for the present study. Although the assumptions seem reasonable, it would be worth considering checking their validity before undertaking further studies. In terms of priorities, the heat balance for the lake indicates that the thermal regime is dominated by heat and mass transfers between the water surface and the atmosphere; heat fluxes accompanying stream inflows and seepage outflow seem small in comparison. Hence, if further field work were to be undertaken, it should probably focus firstly on improving information on meteorology, and secondly on inflows and outflows. For the meteorology, assuming the Te Rokotai weather station is to remain at its present location, consideration could be given to checking the relation between conditions measured at the station versus those over the lake. For the inflows, consideration could be given to installing a temperature logger in the inflow stream, and possibly to measuring discharge. Further investigations could focus on groundwater dynamics.

Although the inflows and outflows may play a minor role in the lake's heat balance and in the patterns of temperature stratification and mixing, they almost certainly play a major role in the nutrient budget of the lake. If modeling of water quality were to be contemplated for the future using a process-based aquatic ecosystem model that coupled with the stratification and mixing model used in this study, monitoring of inflow chemistry and flows would need to be undertaken.

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